Kelly Criterion Research

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# Introduction

<https://en.wikipedia.org/wiki/Kelly_criterion>

Summary

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | Two State | Discrete | Continuous | Log-Normal Distribution |
| One variable | [1, T](#_One_Variable,_Two-State) | [1, D](#_One_Variable,_Multi-State) | [1, C](#_One_Variable,_probability) | [1, Log-Normal](#_One_variable,_Log-normal) |
| Two variables | [2, T](#_Two_variables,_two) | [2, D](#_Two_variables,_Multi-State) | [2, C](#_Two_Variable,_probability) | 2, Log-Normal |
| Multi-Variables | n, T | n, D | n, C | n, Log-Normal |

Variable definition:

|  |  |
| --- | --- |
|  | Gain in log scale, means value doubled, means value half |
|  | Gamely percentage rate of return (i.e. APR) |
|  | Gamely log rate of return |

Notation:

|  |  |
| --- | --- |
|  | Probability of winning |
|  | Probability of losing |
|  | Odds if win |
|  | Odds if lose |
|  | Total assets at day(round) |
|  | Ratio of investment |

## 

# One Random Variable

## Two-States

This is the most simple and standard Kelly Criterion.

Recursion formula:

After games:

Rearrange:

The left-hand side is the average return rate of each game, we want it to be maximum.

Take logarithm of both side:

Take derivative and let it equal to 0:

The second order derivative is:

This proves monotonically decreasing.

This means we should investment only if .

If , then we should put all money to that investment.

The max return rate is where :

If we use to replace, we get:

## Multi-States

Instead of two-state, the discrete probability function can be furthermore represented as:

The formula can be then written as:

The still proves under multi-stages the mono decrease. This will be helpful on computation of the zero point.

If we use to replace, we get:

## Probability Density Function

Rewrite above equation to integral format:

If we use to replace, we get:

# 

## Normal Distribution

This is a special case where the probability density function is normal distribution.

If we use to replace, we get:

# 

# Two Independent Random Variables

## Two States

After games:

## Multi-State (discrete probability function)

## Probability density function

If we use to replace, we get:

## Normal distribution for density function

If we use to replace, we get:

## More variable, probability density function

## Log, More variable, Normal distribution for density function

Since monotonically decreasing for every , the task is then finding a to satisfy :

# Consider Correlation

## Two variables

Hessian Matrix:

# What if *P* is changing

Assuming there will be two probability distribution occur for the same random variable equally:

After games:

Rearrange:

The left-hand side is the average return rate of each game, we want it to be maximum.

Take logarithm of both side:

Take derivative and let it equal to 0:

# Summary

If an investment has , then no need to contribute to that.